1A Ratio, Proportion & Partnership

THEORY

Ratio

- A ratio is a fraction (either proper or improper) which compares two or more quantities of similar kind, which enables us to understand as to how many times one quantity is involved in the other.

- If A : B ($\frac{A}{B}$) is a ratio, then the numerator A is called "Antecedent" and the denominator B is called the "Consequent".

- Ratios must be expressed in the simplest possible form and we can calculate ratios only when the quantities are commensurable (fully quantifiable).

- Two or more ratios can be bridged in order to have a continuous comparison between more than two variables.

- Rule for bridging more than two ratios :

  If ,a,b,c,d,e are five Quantities, and

$$\frac{a}{b}=\frac{N_1}{D_1};\frac{b}{c}=\frac{N_2}{D_2};\frac{c}{d}=\frac{N_3}{D_3};\frac{d}{e}=\frac{N_4}{D_4}$$

Then, a:b:c:d:e= $N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Let a : b is a ratio, then:

- $\frac{a}{b}>1$ (Ratio of Greater Inequality)
- $\frac{a}{b}<1$ (Ratio of Lesser Inequality)
- $\frac{a}{b}=1$ (Ratio of Equality)

- $a^2 : b^2$ (Duplicate Ratio)
- $a^3 : b^3$ (Triplicate Ratio)
- $\sqrt{a} : \sqrt{b}$ (Sub-Duplicate Ratio)
- $\sqrt[3]{a} : \sqrt[3]{b}$ (Sub-Triplicate Ratio)
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$ If then the value of each ratio can be obtained by mean of any one of the following two operations;

a. Each ratio = $\frac{a+c+e+\ldots.}{b+d+f+\ldots.}$ (ADDENDO)

Or

b. Each ratio = $\frac{a-c-e-\ldots.}{b-d-f-\ldots.}$ (SUBTRANDENDO)

INVERSE RATIO:

- IR of a:b is b : a
- IR of a:b:c is bc : ac : ab
- IR of a:b:c:d is bcd : acd : abd : abc

COMPOUND RATIO:

The multiplying effect of all ratios given is known as compound ratio. If a:b and c:d are two ratios, then ac : bd is called the compounded ratio of the two.

Proportion

- Proportion is defined as the equality of two or more ratios. If $\frac{a}{b} = \frac{c}{d}$, in such a case the quantities a,b,c,d are said to be proportional, here 'd' is called the fourth proportional.
- If $\frac{a}{b} = \frac{b}{c}$, then a,b,c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.
- If $\frac{a}{b} = \frac{b}{c}$ or b^2 = ac $\therefore$ b = $\sqrt{ac}$

IF	THEN	PROPERTY
$\frac{a}{b}=\frac{c}{d}$	ad = bc	PRODUCT OF EXTREMES = PRODUCT OF MEANS
	$\frac{b}{a}=\frac{d}{c}$	INVERTENDO
	$\frac{a}{c}=\frac{b}{d}$	ALTERNENDO
	$\frac{a+b}{b}=\frac{c+d}{d}$	COMPONENDO
	$\frac{a-b}{b}=\frac{c-d}{d}$	DIVIDENDO
	$\frac{a+b}{a-b}=\frac{c+d}{c-d}$	COMPONENDO & DIVIDENDO
	$\frac{a-b}{a+b}=\frac{c-d}{c+d}$	DIVIDENDO & COMPONENDO

1B Indices, Surds and Logarithms

THEORY

$a^x = N$

a = base

x = Power/Exponent/Index

N = Product

[But, $a \neq 0, 1, \pm\infty$]

Theory of Indices deals with the various changes in power, during various mathematical operations.

Basic Rules

1. $a^m \times a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$; m is added n times

4. $(ab)^m = a^m x b^m$

5. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

6. $a^0 = 1$

7. $a^{-n} = \dfrac{1}{a^n}$

8. If $a^m = a^n \Rightarrow m = n$; where, $a \neq 0, 1, -1, \pm\infty$

9. For $a^m = b^m$ if m $\neq 0$ then

(i) $a = b$ (when m is odd)

(ii) $a = \pm b$ (when m is even)

10. $a^x = N$

$\Rightarrow a = N^{\frac{1}{x}} = \sqrt[x]{N}$

11. $(i) 0^a = 0$

$(ii) 1^a = 1$

$(iii) a^1 = a$

$(iv) a^0 = 1$

$(v) 0^0$ *has* no meaning

Basic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a+b)(a-b)$

4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

5. $(a+b)^2 - (a-b)^2 = 4ab$

6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

7. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

8. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

11. If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

12. If $a^3+b^3+c^3=3abc$, then either $a+b+c=0$ or $a=b=c$
but both the results cannot hold true simultaneously

Rational Numbers, Irrational Numbers & Surds

- A Rational Number is a number which can be expressed in the form p/q, where q ≠ 0; p & q are integers and p and q are prime to each other, i.e., there is no common factor between p & q, other than 1.
- Any terminating and recurring decimals are rational numbers.
- Thus any non-recurring and non-terminating decimals are irrational numbers, and when the irrational numbers are expressed in radical form (root form), it is known as "Surds".
- Thus all the surds are irrational, but all irrational numbers are not surds.
- The numbers whose perfect root can be evaluated are rational quantities and numbers for which perfect roots cannot be evaluated are irrational quantities.

Order of Surds

If $\sqrt[k]{m}=(m)^{\frac{1}{k}}$ is a surd, then, it is said to be a surd of order "k".

Pure Surds and Mixed Surds

In case of pure surds, entire expression is kept within the radical sign. In mixed surds, it is expressed as a product of one rational and one irrational quantity.

Example:
$\sqrt{7}$ is a pure surd; $\sqrt{12}=\sqrt{4x3}=2\sqrt{3}$ is a mixed surd.

Conjugate of a Surd

If $(a+\sqrt{b})$ or $(\sqrt{a}+\sqrt{b})$ are surds, their respective conjugates would be given by,

$(a-\sqrt{b})$ or $(\sqrt{a}-\sqrt{b})$ and vice-versa.

Rationalization of Surds

Rationalization is a process, where we convert the irrational part of the surd into a rational quantity, with help of its conjugate.

Note: 1

- Rational + Rational = Rational
- Rational − Rational = Rational
- Rational x Rational = Rational
- Rational ÷ Rational = Rational

Note: 2

- Irrational + Irrational = Irrational
- Irrational − Irrational = Rational (only when the quantities are equal); otherwise −
- Irrational − Irrational = Irrational
- Irrational x Irrational = May be Rational or Irrational
- Irrational ÷ Irrational = May be Rational or Irrational

Note: 3

- Rational + Irrational = Irrational
- Rational − Irrational = Irrational
- Rational x Irrational = Irrational
- Rational ÷ Irrational = Irrational

Square Root of Surds

- The square root of a surd is always a surd.

- Every answer for square root must contain +ve or −ve sign and in the absence of +/- sign, "none of these" will be marked as answer.

- If the given surd, whose square root is to be evaluated is in the form $\left(a \pm \sqrt{b}\right)$, then the answer will also be in the form $\pm\left(x \pm \sqrt{y}\right)$.

- Square the options, in order to get the question back.

Logarithms

THEORY

If $a^x=N$, then $x=\log_a N$; * $a \neq 0,1, \pm\alpha$ and for the purpose of log, any negative quantity.

* x is called the logarithm of N (product) to the base "a".

Base "a"

- The base "a" of log can be any positive real number except 1.
- The base of log can be clearly divided into two parts:
- 0 < a < 1 (the proper fraction)
- a > 1 (positive integer / mixed fraction)
- Unless otherwise specified, the base of log is always taken to be 10 and this is known as Common Logarithm.
- For theoretical purpose, the base is always taken to be "e", where "e" is a constant and this is known as "Natural Logarithm".
- Common Logarithms are used for numerical calculations and Natural Logarithms are used in calculus.

Basic Rules

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a 0 =$ Undefined
7. $\log_a -ve =$ Undefined

8. $\log_a m = \log_a n \Rightarrow m = n$

Change of Base in Logarithms

1. $\log_b a = \dfrac{\log_m a}{\log_m b}$ (m can be any common base) ($m \neq 0, 1, \pm\alpha$,-ve value)

2. $\log_a b = \dfrac{1}{\log_b a}$

3. $a^{\log_a x} = x$

Nature of Log Values

- All the values which are obtained from log tables are irrational numbers provided the numbers are not 10 or in the form of 10^n.
- $\log_b a$ is a rational quantity only when, $\dfrac{\log a}{\log b}$ is rational.
- If K is a number, then its log value, logK can be divided into two parts: a) Integral Part, b) Fractional Part.
- The integral part is called "Characteristics" and the fractional part is called "Mantissa".
- The integral characteristics part can be positive or negative or zero but not a fraction.
- The values of mantissa are always positive fractions.
- The values for mantissa are obtained from log tables.
- Characteristics are to be calculated before we evaluate mantissa from the log table.
- Value of characteristics = number of significant digits before decimal – 1